LaTeX to PDF and MathJax: Example 2

## LaTeX to PDF and MathJax: Example 2

Emma Cliffe

2017

### [Contents](#x1-1000)

1 [The scalar product](#x1-40001)
 1.1 [Vectors in cartesian form](#x1-50001.1)
2 [Using Matlab](#x1-60002)

### [List of Figures](#x1-2000)

1 [Two vectors with angle between them.](#x1-40011)
2 [A triangle is formed by two vectors and their difference.](#x1-50022)

### [Using this document](#x1-3000)

This is a second example of a document compiled from LATEX  into multiple formats.

* [Standard print PDF](https://stem-enable.github.io/LaTeXtoPDFandMathJax-Example2/LaTeXtoPDFandMathJax-2-standard.pdf)
* [Clearer print PDF](https://stem-enable.github.io/LaTeXtoPDFandMathJax-Example2/LaTeXtoPDFandMathJax-2-clear.pdf)
* [Accessible web format](https://stem-enable.github.io/LaTeXtoPDFandMathJax-Example2/)
* [Accessible Word document](https://stem-enable.github.io/LaTeXtoPDFandMathJax-Example2/LaTeXtoPDFandMathJax-2.docx)

The outputs can be used to test setups and as a second example for students to try out.

### 1 [The scalar product](#QQ2-1-4)

Consider two vectors $a$ and $b$ drawn so their tails are at the same point.



Figure 1: Two vectors with angle between them.

We define the scalar product of $a$ and $b$ as follows.

 Definition 1.1 (Scalar product).
The scalar product of $a$ and $b$ is

$$\left.a⋅b=|a||b|cosθ\right.$$

where

* $\left|a\right|$ is the modulus of $a$,
* $\left|b\right|$ is the modulus of $b$, and
* $θ$ is the angle between $a$ and $b$.

 Remark 1.2.
It is important to use the dot symbol for the scalar product (also called the dot product). You must not use a $×$ symbol as this denotes the vector product which is defined differently.

 Example 1.3.
Let

$$a=\left(\begin{matrix}2\\2\end{matrix}\right) and b=\left(\begin{matrix}4\\0\end{matrix}\right).$$

The angle between these vectors is $θ=45^{∘}$. Then $\left|a|=\sqrt{8}\right.$ and $\left|b|=4\right.$. So,

$$\begin{matrix}a⋅b=\left(\begin{matrix}2\\2\end{matrix}\right) ⋅ \left(\begin{matrix}4\\0\end{matrix}\right)&\left.=|a||b|cosθ  \right.&&  \\ &=\sqrt{8}×4×cos45^{∘}  &&  \\ &=4\sqrt{8}×\frac{1}{\sqrt{2}}=4\frac{\sqrt{8}}{\sqrt{2}}=4\sqrt{4}=8.  &&  \end{matrix}$$

Note that the result is a scalar, not a vector.

#### 1.1 [Vectors in cartesian form](#QQ2-1-6)

When vectors are given in cartesian form there is an alternative formula for calculating the scalar product.

 Proposition 1.4.
If $a=a\_{1}i+a\_{2}j$ and $b=b\_{1}i+b\_{2}j$ then

$$a⋅b=a\_{1}b\_{1}+a\_{2}b\_{2}.$$

Proof.  Consider the vector $b−a=\left(\begin{matrix}b\_{1}−a\_{1}\\b\_{2}−a\_{2}\end{matrix}\right)$. The modulus of this is

$$\left|b−a|=\sqrt{\left(b\_{1}−a\_{2}\right)^{2}+\left(b\_{2}−a\_{2}\right)^{2}}.\right.$$

Note from figure [2](#x1-50022) that the vectors $a$, $b$ and $b−a$ form a triangle:



Figure 2: A triangle is formed by two vectors and their difference.

Let $θ$ denote the angle between $a$ and $b$. Then, the cosine rule yields:

|  |  |
| --- | --- |
| $$\left.\left|b−a\right|^{2}=\left|a\right|^{2}+\left|b\right|^{2}−2|a||b|cosθ.\right.$$ | (1) |

Substituting the definition of the scalar product of $a$ and $b$ into equation [1](#x1-5003r1) gives:

$$\left|b−a\right|^{2}=\left|a\right|^{2}+\left|b\right|^{2}−2\left(a⋅b\right).$$

Rearranging:

$$\left(a⋅b\right)=\frac{1}{2}\left(\left|a\right|^{2}+\left|b\right|^{2}−\left|b−a\right|^{2}\right).$$

Writing this in terms of components produces:

$$\begin{matrix}\left(a⋅b\right)&=\frac{1}{2}\left(a\_{1}^{2}+a\_{2}^{2}+b\_{1}^{2}+b\_{2}^{2}−\left(b\_{1}−a\_{1}\right)^{2}−\left(b\_{2}−a\_{2}\right)^{2}\right)  &&  \\ &=\frac{1}{2}\left(a\_{1}^{2}+a\_{2}^{2}+b\_{1}^{2}+b\_{2}^{2}−b\_{1}^{2}+2b\_{1}a\_{1}−a\_{1}^{2}−b\_{2}^{2}+2b\_{2}a\_{2}−a\_{2}^{2}\right)  &&  \\ &=\frac{1}{2}\left(2b\_{1}a\_{1}+2b\_{2}a\_{2}\right)  &&  \\ &=a\_{1}b\_{1}+a\_{2}b\_{2}  &&  \end{matrix}$$

as required. □

 Example 1.5.
Consider again the vectors

$$a=\left(\begin{matrix}2\\2\end{matrix}\right) and b=\left(\begin{matrix}4\\0\end{matrix}\right).$$

Calculating the scalar product using the components:

$$a⋅b=a\_{1}b\_{1}+a\_{2}b\_{2}=2×4+2×0=8.$$

Note that if we are given vectors in this form, the scalar product may be used to calculate the angle between them. Since $a⋅b=8$ and we have:

$$\begin{matrix}\left|a\right|&=\sqrt{8}  &&  \\\left|b\right|&=4.  &&  \end{matrix}$$

Hence,

$$\begin{matrix}8=a⋅b&\left.=|a||b|cosθ  \right.&&  \\ &=4\sqrt{8}cosθ.  &&  \end{matrix}$$

Rearranging:

$$θ=cos^{−1}\left(\frac{8}{4\sqrt{8}}\right)=45^{∘}.$$

### 2 [Using Matlab](#QQ2-1-8)

Two calculate the scalar product in Matlab the dot function is used.

Create two vectors:

> A = [4 -1 2];
> B = [2 -2 -1];

Calculate the scalar product:

> C = dot(A,B)

    C = 8